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Deformations of the Almheiri-Polchinski model

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ABSTRACT: We study deformations of the Almheiri-Polchinski (AP) model by employing the Yang-Baxter deformation technique. The general deformed AdS_2 metric becomes a solution of a deformed AP model. In particular, the dilaton potential is deformed from a simple quadratic form to a hyperbolic function-type potential similarly to integrable deformations. A specific solution is a deformed black hole solution. Because the deformation makes the spacetime structure around the boundary change drastically and a new naked singularity appears, the holographic interpretation is far from trivial. The Hawking temperature is the same as the undeformed case but the Bekenstein-Hawking entropy is modified due to the deformation. This entropy can also be reproduced by evaluating the renormalized stress tensor with an appropriate counter-term on the regularized screen close to the singularity.

KEYWORDS: 2D Gravity, Integrable Field Theories, AdS-CFT Correspondence, Black Holes

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1 Introduction

In the recent study of string theory, one of the most important issues is to understand a holographic principle [1, 2] at the full quantum level (For a review see [3]). The AdS/CFT correspondence [4–6] is a realization of the holography. This is, however, a conjectured relation and there is no rigorous proof so far. The integrable structure behind the correspondence at the planar level has played an important role in checking conjectured relations in non-BPS regions (For a comprehensive review, see [7]). But the proof is still far from the completion and furthermore it does not seem likely that the integrability would work in the presence of a black hole.

Towards the complete understanding of holography, it is significant to try to construct a simple toy model of quantum holography. In fact, Kitaev proposed such a model [8], which is a variant of the Sachdev-Ye (SY) model [9]. More concretely, this model is a one-dimensional system composed of $N \gg 1$ fermions with a random, all-to-all quartic interaction. This model is now called the Sachdev-Ye-Kitaev (SYK) model.¹ It should be remarked that the Lyapunov exponent computed from an out-of-time-order four-point function [19, 20] in the SYK model saturates the bound presented in [21]. This is the onset to open a window to a toy model of holography because the Lyapunov exponent of black

¹For the recent progress on the SYK model, see [10, 11, 13–18].

hole in Einstein gravity is $2\pi/\beta$ [22, 23], where β is the inverse of the Hawking temperature (For a black hole S-matrix approach, see [24]).

A promising candidate of the gravity dual for the SYK model is a 1+1 D dilaton gravity. This system was originally introduced by Jackiw [25] and Teitelboim [26] (For a nice review of the 1+1 D dilaton gravity system, see [27]). From a renewed interest, the dilaton gravity with a certain dilaton potential was intensively studied in the recent work [28], and this model is called the Almheiri-Polchinski (AP) model. A black hole solution exists as a vacuum solution of the AP model. They studied its various properties like the RG flow structure at zero temperature, the Bekenstein-Hawking entropy, the renormalized boundary stress tensor, and the contribution of conformal matter fields to the entropy. For the recent progress on the AP model, see [29, 30].

In this paper, we are concerned with deformations of the AP model. Why is it so interesting to study the deformations? There are some observation and motivation based on the recent progress. The first is an observation that the SY model is constructed by performing a disordered quench to an *isotropic* quantum Heisenberg magnet [9]. The Heisenberg model itself is integrable. Hence, supposing that the conjectured duality is true, it is natural to expect that integrable deformations of it lead to the associated deformations of the AP model. The second is a motive to understand the holographic duals of deformed AdS_2 geometries. Recently, a systematic way to perform integrable deformations, which is called the Yang-Baxter deformation [31–34],² has been intensively studied.³ However, the holographic duals of the deformed geometries have been poorly understood. In particular, even the location of the holographic screen has not been clarified, though there is a proposal [69–71]. Hence it is important to get much deeper understanding of the simplest case like AdS_2 . Furthermore, the Yang-Baxter deformation is not applicable to black hole geometries in general, because those cannot be described usually as a coset, homogeneous space. However, it is not the case for a 1+1 D black hole presented in this paper.

Based on the observation and motivation described above, we will study deformations of the AP model by employing the Yang-Baxter deformation technique. The general deformed AdS_2 metric becomes a solution of a deformed AP model. In particular, the dilaton potential is deformed from a simple quadratic form to a hyperbolic function-type potential similarly to integrable deformations. A specific solution is a deformed black hole solution. Because the deformation makes the spacetime structure around the boundary change drastically and a new naked singularity appears, the holographic interpretation is far from trivial. The Hawking temperature is the same as the undeformed case but the Bekenstein-Hawking entropy is modified due to the deformation. This entropy can also be reproduced by evaluating the renormalized stress tensor with an appropriate counter-term on the regularized screen close to the singularity.

This paper is organized as follows. In section 2 we study the most general Yang-Baxter deformation of AdS_2 . Section 3 introduces the classical action of 1+1 D dilaton gravity and the AP model as a special case. In section 4 we study deformations of the AP model. The

²For the (affine) symmetry algebras, see [35–39] and [40–42].

³For Yang-Baxter deformations of type IIB superstring on $\text{AdS}_5 \times S^5$, see [43–68].

most general deformed metric constructed in section 2 becomes a solution with a deformed dilaton potential. This deformed system allows a black hole solution as a specific solution like in the AP model. The Bekenstein-Hawking entropy is also computed. In section 5, we revisit the black hole entropy from the viewpoint of the renormalized boundary stress tensor. Putting a regularized screen close to a singularity, we evaluate the renormalized boundary stress tensor with an appropriate counter-term. The resulting entropy nicely agrees with the Bekenstein-Hawking entropy computed in section 4. Section 6 is devoted to conclusion and discussion.

2 Yang-Baxter deformations of AdS_2

In this section, we consider the most general Yang-Baxter deformation of the AdS_2 metric. First of all, we briefly describe a coset construction of the Poincaré AdS_2 . Then we study the most general Yang-Baxter deformation of Poincaré AdS_2 . As a result, we obtain a three-parameter family of deformed AdS_2 spaces.

2.1 Coset construction of AdS_2

Let us recall a coset construction of the Poincaré AdS_2 metric (For the detail of the coset construction, for example, see [72]).

The starting point is that the AdS_2 geometry is represented by a coset

$$\text{AdS}_2 = \text{SL}(2)/\text{U}(1). \quad (2.1)$$

By using the coordinates t and z , a coset representative g is parametrized as

$$g = \exp[tH] \exp[(\log z)D], \quad (2.2)$$

where H and D are the time translation and dilatation generators, respectively. By involving the special conformal generator C , the $\mathfrak{sl}(2)$ algebra in the conformal basis is spanned as

$$[D, H] = H, \quad [C, H] = 2D, \quad [D, C] = -C. \quad (2.3)$$

These generators can be represented by the $\mathfrak{so}(1, 2)$ ones T_I ($I = 0, 1, 2$) like

$$H \equiv T_0 + T_2, \quad C \equiv T_0 - T_2, \quad D \equiv T_1, \quad (2.4)$$

where T_I 's satisfy the commutation relations:

$$[T_0, T_1] = -T_2, \quad [T_1, T_2] = T_0, \quad [T_2, T_0] = -T_1. \quad (2.5)$$

In the following, we will work with T_I 's in the fundamental representation,

$$T_0 = \frac{i}{2}\sigma_1, \quad T_1 = \frac{1}{2}\sigma_2, \quad T_2 = \frac{1}{2}\sigma_3, \quad (2.6)$$

where σ_i ($i = 1, 2, 3$) are the standard Pauli matrices.

Note here that the coset (2.1) is symmetric as one can readily understand from (2.5). When vector spaces \mathfrak{h} and \mathfrak{m} are spanned as

$$\mathfrak{h} = \text{span}_{\mathbb{R}} \langle T_2 \rangle, \quad \mathfrak{m} = \text{span}_{\mathbb{R}} \langle T_0, T_1 \rangle,$$

the \mathbb{Z}_2 -grading structure is expressed as

$$[\mathfrak{h}, \mathfrak{h}] \subset \mathfrak{h}, \quad [\mathfrak{m}, \mathfrak{h}] \subset \mathfrak{m}, \quad [\mathfrak{m}, \mathfrak{m}] \subset \mathfrak{h}. \quad (2.7)$$

When representing the $\mathfrak{sl}(2)$ algebra by a direct product (as vector spaces),

$$\mathfrak{sl}(2) = \mathfrak{h} \oplus \mathfrak{m},$$

the projection operator $P : \mathfrak{sl}(2) \rightarrow \mathfrak{m}$ can be defined as

$$P(X) \equiv \frac{\text{Tr}(X T_0)}{\text{Tr}(T_0 T_0)} T_0 + \frac{\text{Tr}(X T_1)}{\text{Tr}(T_1 T_1)} T_1, \quad X \in \mathfrak{sl}(2). \quad (2.8)$$

Now the Poincaré AdS_2 metric can be computed by performing coset construction. The left invariant one-form $J = g^{-1}dg$ is expanded as

$$J = e^0 T_0 + e^1 T_1 + \frac{1}{2} \omega^{01} T_2.$$

Here e^0 and e^1 are the zweibeins, and ω^{01} is the spin connection. With the parametrization (2.2), the zweibeins are given by

$$e^0 = \frac{dt}{z}, \quad e^1 = \frac{dz}{z}.$$

By using the projection operator P in (2.8) and the explicit expressions of the zweibeins e^0 and e^1 , the resulting metric is obtained as

$$\begin{aligned} ds^2 &= 2\text{Tr}[JP(J)] = -e^0 e^0 + e^1 e^1 \\ &= \frac{-dt^2 + dz^2}{z^2}. \end{aligned} \quad (2.9)$$

This is nothing but the AdS_2 metric in the Poincaré coordinates.

Hereafter, it is often convenient to use the the light-cone coordinates defined as

$$x^{\pm} \equiv t \pm z. \quad (2.10)$$

Then the metric is rewritten as

$$ds^2 = -e^{2\omega(x^+, x^-)} dx^+ dx^- = -\frac{4dx^+ dx^-}{(x^+ - x^-)^2}. \quad (2.11)$$

The exponential factor will play an important role in later discussion.

2.2 The general Yang-Baxter deformation

Let us next consider Yang-Baxter deformations of the AdS_2 metric (2.9). In the usual discussion, Yang-Baxter deformations [31–34] are performed for 2D non-linear sigma models. Then the anti-symmetric two-form is also involved as well as the metric. Here we will concentrate on the metric part only.

The prescription of the deformation is very simple. It is just to insert a factor as follows:

$$ds^2 = 2\text{Tr} \left[J \frac{1}{1 - 2\eta R_g \circ P} P(J) \right]. \quad (2.12)$$

Here η is a constant parameter which measures the deformation. Then R_g is defined as a chain of operation like

$$R_g(X) \equiv g^{-1} \circ R(gXg^{-1}) \circ g, \quad (2.13)$$

where g is the group element in (2.2). The key ingredient is a linear operator $R: \mathfrak{sl}(2) \rightarrow \mathfrak{sl}(2)$, and satisfy the (modified) classical Yang-Baxter equation [(m)CYBE]:

$$[R(X), R(Y)] - R([R(X), Y] + [X, R(Y)]) = c \cdot [X, Y] \quad (X, Y \in \mathfrak{sl}(2)). \quad (2.14)$$

Here c is a real constant parameter. The case with $c \neq 0$ is the mCYBE and the case with $c = 0$ is the homogeneous CYBE.

We consider the most general deformations with the following R -operator

$$R(T_I) = \tilde{\Omega}_{IJ} M^{JK} T_K, \quad (2.15)$$

where $\tilde{\Omega}_{IJ}$ and M^{IJ} are defined as⁴

$$\tilde{\Omega}_{IJ} \equiv \text{Tr}(T_I T_J) = \frac{1}{2} \eta_{IJ}, \quad M^{IJ} \equiv \begin{pmatrix} 0 & m_1 & m_2 \\ -m_1 & 0 & m_3 \\ -m_2 & -m_3 & 0 \end{pmatrix}. \quad (2.16)$$

Putting the ansatz (2.15) into the (m)CYBE (2.14) leads to an algebraic relation,

$$-m_1^2 - m_2^2 + m_3^2 = 4c. \quad (2.17)$$

After evaluating the expression (2.12) with the general ansatz (2.15), one can obtain the following metric:

$$ds^2 = \frac{-dt^2 + dz^2}{z^2 - \eta^2 (\alpha + \beta t + \gamma(-t^2 + z^2))^2}. \quad (2.18)$$

Here α , β and γ are defined as linear combinations of m_p ($p = 1, 2, 3$) as follows:

$$\alpha \equiv \frac{1}{2}(m_1 + m_3), \quad \beta \equiv -m_2, \quad \gamma \equiv \frac{1}{2}(m_1 - m_3). \quad (2.19)$$

⁴A similar study was done for the Nappi-Witten model [73] in [74]. Note that the present definition of M^{IJ} is slightly different from the one in [74].

When $\eta = 0$, the undeformed metric (2.9) is reproduced. Note here that the four constant parameters m_p ($p = 1, 2, 3$) and c appear in our discussion. Then a constraint (2.17), which comes from the (m)CYBE, is imposed. Hence, three of them are independent each other.

The Ricci scalar curvature of the metric (2.18) is

$$R = -2(1 - \tilde{\omega}\eta^2) \frac{z^2 + \eta^2(\alpha + \beta t + \gamma(-t^2 + z^2))^2}{z^2 - \eta^2(\alpha + \beta t + \gamma(-t^2 + z^2))^2}, \quad (2.20)$$

where we have introduced a new quantity,

$$\begin{aligned} \tilde{\omega} &\equiv \beta^2 + 4\alpha\gamma = m_1^2 + m_2^2 - m_3^2 \\ &= -4c. \end{aligned} \quad (2.21)$$

At the last equality, the (m)CYBE (2.14) has been utilized. The scalar curvature (2.20) changes (even its sign) depending on the values of parameters and coordinates, while it becomes a constant -2 in the undeformed limit $\eta \rightarrow 0$. The expression (2.20) indicates that the deformed geometry contains both AdS and dS in general.

3 A brief review of the AP model

In this section, we shall introduce the classical action of 1+1 D dilaton gravity system. Then we briefly describe the AP model and its properties related to our later discussion.

3.1 1+1 D dilaton gravity system

The dilaton gravity system in 1+1 dimensions is composed of the metric g_{ab} ($a, b = 0, 1$) and the dilaton Φ . The coordinates are parametrized as $x^a = (x^0, x^1) = (t, z)$.

The classical action S is given by

$$\begin{aligned} S &= S_{g,\Phi} + S_{\text{matter}}, \\ S_{g,\Phi} &= \frac{1}{16\pi G} \int d^2x \sqrt{-g} (\Phi^2 R - U(\Phi)), \\ S_{\text{matter}} &= \frac{1}{32\pi G} \int d^2x \sqrt{-g} \Omega(\Phi) (\nabla f)^2. \end{aligned} \quad (3.1)$$

Here G is the Newton constant in 1+1 dimensions and $U(\Phi)$ is the dilaton potential.

In the following, we will work with the metric in the conformal gauge,

$$ds^2 = -e^{2\omega(x^+, x^-)} dx^+ dx^-, \quad (3.2)$$

where the light-cone coordinates are defined in (2.10).

Then the equations of motion are given by

$$\begin{aligned} \partial_+(\Omega\partial_-f) + \partial_-(\Omega\partial_+f) &= 0, \\ 4\partial_+\partial_-\Phi^2 - e^{2\omega}U(\Phi) &= 0, \\ 2\partial_+(e^{-2\omega}\partial_-e^{2\omega}) - \frac{1}{2}e^{2\omega}\partial_{\Phi^2}U(\Phi) &= (\partial_{\Phi^2}\Omega)\partial_+f\partial_-f, \\ -e^{2\omega}\partial_+(e^{-2\omega}\partial_+\Phi^2) &= \frac{\Omega}{2}\partial_+f\partial_+f, \\ -e^{2\omega}\partial_-(e^{-2\omega}\partial_-\Phi^2) &= \frac{\Omega}{2}\partial_-f\partial_-f. \end{aligned} \quad (3.3)$$

The energy-momentum tensor for the matter field f is normalized as

$$\begin{aligned}(T_{\text{matter}})_{ab} &\equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{matter}}}{\delta g^{ab}} \\ &= -\frac{\Omega(\Phi)}{16\pi G} \left(\partial_a f \partial_b f - \frac{1}{2} g_{ab} \partial^c f \partial_c f \right).\end{aligned}\quad (3.4)$$

This expression (3.4) is valid for the general form of $\Omega(\Phi)$.

3.2 The AP model

The AP model corresponds to a special case of 1+1 D dilaton gravity specified by the following condition:⁵

$$U(\Phi) = 2 - 2\Phi^2, \quad \Omega(\Phi) = 1. \quad (3.5)$$

This model exhibits nice properties. Among them, we are concerned with the vacuum solution of this model. For our later convenience, we shall give a brief review of the work [28] by focusing upon the vacuum solution in the following.

The general vacuum solution is given by

$$ds^2 = \frac{1}{z^2} (-dt^2 + dz^2), \quad (3.6)$$

$$\Phi^2 = 1 + \frac{a + bt + c(-t^2 + z^2)}{z}, \quad (3.7)$$

and depends on three real constants a , b and c . This three-parameter family contains interesting solutions as specific examples. For example, the case with $a = 1/2$, $b = 0$ and $c = 0$ corresponds to a renormalization group flow solution from a conformal Lifshitz spacetime to AdS_2 [28], with an appropriate lift-up to higher dimensions.

Another intriguing example is a black hole solution specified with $a = 1/2$, $b = 0$ and $c = \mu/2$, where μ is a real positive constant. Then by performing a coordinate transformation,

$$x^\pm = \frac{1}{\sqrt{\mu}} \tanh(\sqrt{\mu}(T \pm Z)), \quad (3.8)$$

the solution is rewritten into the following form:

$$\begin{aligned}ds^2 &= \frac{4\mu}{\sinh(2\sqrt{\mu}Z)} (-dT^2 + dZ^2), \\ \Phi^2 &= 1 + \sqrt{\mu} \coth(2\sqrt{\mu}Z).\end{aligned}\quad (3.9)$$

The new coordinates T and Z cover a smaller region which is in the inside of the entire Poincaré AdS_2 , as depicted in figure 1.

⁵This system is basically the same as the Jackiw-Teitelboim model [25, 26] up to a topological term. We appreciate D. Vassilevich and the referee for elucidating this point.

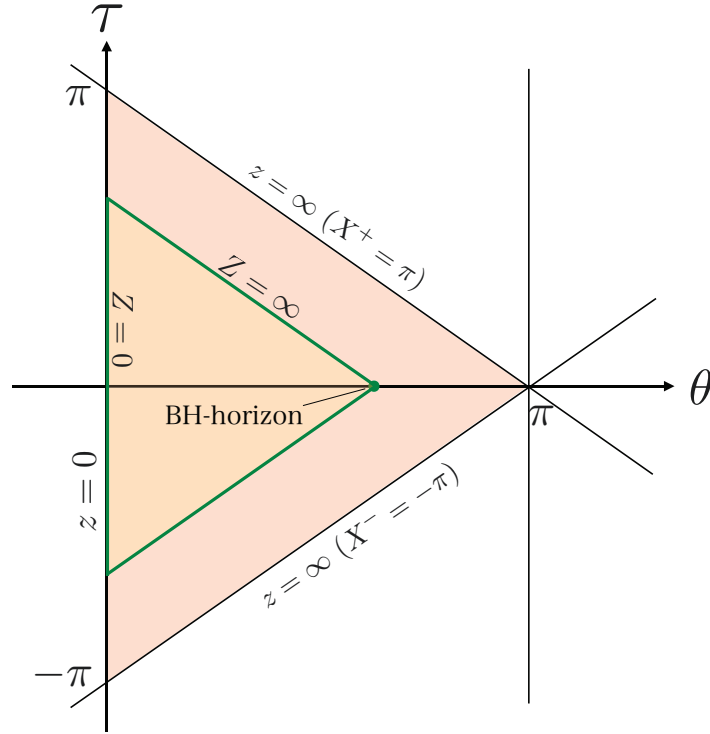


Figure 1. Penrose diagram of the black hole solution. The global AdS_2 is parametrized by τ and θ . The largest red triangle describes the Poincaré patch of AdS_2 as usual. The coordinate system (3.9) covers the inside of smaller triangle bounded by green lines. The right vertex corresponds to the black hole horizon which is specified as the point that T is finite but Z is infinity.

The background (3.9) indeed describes a black hole geometry, but it may not be so manifest. To figure out the black hole geometry, it is nice to move to the Schwarzschild coordinates by performing a further coordinate transformation,

$$Z = \frac{1}{2\sqrt{\mu}} \text{arccoth} \left(\frac{\rho}{\sqrt{\mu}} \right). \quad (3.10)$$

Then the background (3.9) can be rewritten as⁶

$$ds^2 = -4(\rho^2 - \mu)dt^2 + \frac{d\rho^2}{\rho^2 - \mu}, \quad \Phi^2 = 1 + \rho. \quad (3.11)$$

In this metric, the black hole horizon is located at $\rho = \sqrt{\mu}$, and the Hawking temperature T_H can be evaluated in the standard manner as

$$T_H = \frac{1}{4\pi} \partial_\rho \sqrt{\frac{-g_{tt}}{g_{\rho\rho}}} \Big|_{\rho=\sqrt{\mu}} = \frac{\sqrt{\mu}}{\pi}. \quad (3.12)$$

⁶The factor 4 is included so that the Bekenstein-Hawking entropy should match with the holographic computation. This normalization guarantees the matching of the bulk and boundary times (or temperatures). We are grateful to Ahmed Almheiri for this point.

Thus one can see that the background (3.9) describes a black hole whose horizon is located at $Z = \infty$.

The Bekenstein-Hawking entropy can also be computed as

$$S_{\text{BH}} = \frac{A}{4G_{\text{eff}}} \Big|_{Z \rightarrow \infty} = \frac{\Phi^2}{4G} \Big|_{\sqrt{\mu} = \pi T_H} = \frac{1 + \pi T_H}{4G}. \quad (3.13)$$

Here the area A is taken as $A = 1$ because the horizon is just a point, and the effective Newton constant G_{eff} can be read off from the classical action as

$$\frac{1}{G_{\text{eff}}} = \frac{\Phi^2}{G}. \quad (3.14)$$

On the other hand, the holographic entropy can be computed by using the renormalized boundary stress tensor. For the detailed computation like the regularization and the counter-term, see [28]. As a result, the renormalized boundary stress tensor is evaluated as

$$\langle \hat{T}_{tt} \rangle = \frac{\mu}{8\pi G} \equiv E. \quad (3.15)$$

Then by using the thermodynamic relation

$$dS = \frac{dE}{T_H}, \quad (3.16)$$

the entropy is obtained as

$$S = \frac{\pi T_H}{4G} + S_{T_H=0}, \quad (3.17)$$

where $S_{T_H=0}$ is an integration constant. Thus the holographic entropy agrees with the Bekenstein-Hawking entropy, up to the temperature-independent constant.

The main goal of this paper is to realize this correspondence of the entropies for a deformed black hole solution introduced in the next section.

4 Deforming the AP model

In this section, we consider deforming the AP model so that the deformed AdS_2 metric (2.18) is supported as a solution. For simplicity, the matter fields are turned off hereafter. Along this line, as well as the dilaton itself, the dilaton potential also has to be deformed from a simple quadratic one (3.5) to a hyperbolic function, similarly to integrable deformations.

4.1 The deformed AP model

The deformed metric. Before discussing the dilaton and the dilaton potential, it is helpful to rewrite the deformed metric (2.18) as

$$\begin{aligned} ds^2 &= \frac{-dt^2 + dz^2}{z^2 - \eta^2 (\alpha + \beta t + \gamma(-t^2 + z^2))^2} \\ &= \frac{1}{1 - \eta^2 (X \cdot P)^2} \frac{-dt^2 + dz^2}{z^2}. \end{aligned} \quad (4.1)$$

Here we have introduced new quantities: a coordinate vector X^I and a parameter vector P_I defined as

$$\begin{aligned} X^I &\equiv \frac{1}{z} \left(t, \frac{1}{2}(1+t^2-z^2), \frac{1}{2}(1-t^2+z^2) \right), \\ P_I &\equiv (\beta, \alpha - \gamma, \alpha + \gamma) \quad (I, J = 1, 2, 3). \end{aligned} \quad (4.2)$$

The metric of the embedding space $\mathbb{M}^{2,1}$ is taken as $\eta_{IJ} = \text{diag}(-1, +1, -1)$. The inner products are defined as

$$X \cdot P \equiv X^I P_I = \frac{\alpha + \beta t + \gamma(-t^2 + z^2)}{z}, \quad (4.3)$$

$$X \cdot X \equiv \eta_{IJ} X^I X^J = -1, \quad P \cdot P \equiv \eta^{IJ} P_I P_J = -\tilde{\omega}. \quad (4.4)$$

These three products $X \cdot X$, $P \cdot P$ and $X \cdot P$ are transformed as scalars under the $\text{SL}(2, \mathbb{R})$ transformation.⁷ For example, $X \cdot P$ is transformed as $X \cdot P = \tilde{X} \cdot \tilde{P}$, where \tilde{X} and \tilde{P} are new coordinate and parameter vectors, respectively.

Using the $\text{SL}(2, \mathbb{R})$ transformation, we can choose the vector \tilde{P} freely as long as it satisfies the relation $\tilde{P} \cdot \tilde{P} = P \cdot P = -\tilde{\omega}$. Note that only the warped factor of the metric changes like

$$ds^2 = \frac{1}{1 - \eta^2(\tilde{X} \cdot \tilde{P})^2} \frac{-d\tilde{t}^2 + d\tilde{z}^2}{\tilde{z}^2} \quad (4.5)$$

because the rigid AdS_2 part is invariant under the $\text{SL}(2, \mathbb{R})$ transformation.

The dilaton sector. Given the deformed metric (2.18) [or equivalently (4.1)], by solving the equations of motion (3.3) without the matter fields, the dilaton Φ is determined as

$$\begin{aligned} \Phi^2 &= \frac{c_1}{2\eta} \log \left| \frac{z + \eta(\alpha + \beta t + \gamma(-t^2 + z^2))}{z - \eta(\alpha + \beta t + \gamma(-t^2 + z^2))} \right| + c_2 \\ &\equiv \frac{c_1}{2\eta} \log \left| \frac{1 + \eta(X \cdot P)}{1 - \eta(X \cdot P)} \right| + c_2, \end{aligned} \quad (4.6)$$

when the dilaton potential is deformed as⁸

$$U(\Phi) = \begin{cases} -(1 - \tilde{\omega} \eta^2) \frac{c_1}{\eta} \sinh \left[\frac{2\eta}{c_1} (\Phi^2 - c_2) \right] & (\text{for } 1 > |\eta(X \cdot P)|) \\ + (1 - \tilde{\omega} \eta^2) \frac{c_1}{\eta} \sinh \left[\frac{2\eta}{c_1} (\Phi^2 - c_2) \right] & (\text{for } 1 < |\eta(X \cdot P)|) \end{cases}. \quad (4.7)$$

Here c_1 and c_2 are arbitrary constants

⁷This $\text{SL}(2, \mathbb{R})$ transformation is the usual one generated by three transformations, 1) time translation, 2) dilatation and 3) special conformal transformation.

⁸According to an interesting paper [75], this dilaton potential leads to a q -deformation of $\mathfrak{sl}(2)$. This result should be closely related to the Yang-Baxter deformation, e.g., [35–39].

In the undeformed limit $\eta \rightarrow 0$, the dilaton (4.6) is reduced to

$$\Phi^2 = c_2 + c_1 \frac{\alpha + \beta t + \gamma(-t^2 + z^2)}{z},$$

and thus the dilaton (3.7) in the AP model has been reproduced when $c_1 = 1$ and $c_2 = 1$. Remarkably, the three parameters α , β and γ correspond to a , b and c in (3.7), respectively. Similarly, as $\eta \rightarrow 0$, the upper branch of the potential (4.7) reduces to

$$U(\Phi) = 2(c_2 - \Phi^2),$$

while the lower branch vanishes. Thus the dilaton potential of the AP model is reproduced when $c_2 = 1$. In total, the case with $c_1 = c_2 = 1$ is associated with the AP model and hence we will work with $c_1 = c_2 = 1$ hereafter.

The vacuum solution in the deformed AP model. In summary, the deformed AP model is specified by the deformed dilaton potential,

$$U(\Phi) = \begin{cases} -(1 - \tilde{\omega} \eta^2) \frac{1}{\eta} \sinh [2\eta(\Phi^2 - 1)] & (\text{for } 1 > |\eta(X \cdot P)|) \\ + (1 - \tilde{\omega} \eta^2) \frac{1}{\eta} \sinh [2\eta(\Phi^2 - 1)] & (\text{for } 1 < |\eta(X \cdot P)|) \end{cases},$$

and the vacuum solution is given by

$$ds^2 = \frac{1}{1 - \eta^2(X \cdot P)^2} \frac{-dt^2 + dz^2}{z^2}, \quad \Phi^2 = \frac{1}{2\eta} \log \left| \frac{1 + \eta(X \cdot P)}{1 - \eta(X \cdot P)} \right| + 1, \quad (4.8)$$

where

$$X \cdot P = \frac{\alpha + \beta t + \gamma(-t^2 + z^2)}{z}.$$

4.2 A deformed black hole solution

In this subsection, we study a deformed black hole solution contained as a special case of the general vacuum solution (obtained in the previous subsection). This solution can be regarded as a deformation of the black hole solution presented in [28].

In the following, instead of $\tilde{\omega}$, we use a new parameter μ defined as

$$\mu \equiv -\tilde{P} \cdot \tilde{P} = \tilde{\omega} = -4c,$$

so as to make our notation the same as that of the AP model. Here it may be worth noting that the black hole temperature is related to the modification of the CYBE. The zero temperature case corresponds to the homogeneous CYBE and the temperature is measured by negative values of c . Solutions of the mCYBE with negative (positive) c are called the split (non-split) type. The well-known example of the non-split type is the q -deformation of AdS_5 [43, 44], while the split type has gotten little attention. For the recent progress on the split type, see [76, 77]. It may be interesting to seek some connection between black hole geometries and solutions of split type.

By performing the same coordinate transformation as in the undeformed case like

$$x^\pm = \frac{1}{\sqrt{\mu}} \tanh(\sqrt{\mu}(T \pm Z)), \quad (4.9)$$

the deformed black hole solution is obtained as

$$\begin{aligned} ds^2 &= \frac{4\mu}{-\eta^2\mu + (1 - \eta^2\mu) \sinh^2(2\sqrt{\mu}Z)} (-dT^2 + dZ^2), \\ \Phi^2 &= 1 + \frac{1}{2\eta} \log \left| \frac{1 + \eta\sqrt{\mu} \coth(\sqrt{\mu}Z)}{1 - \eta\sqrt{\mu} \coth(\sqrt{\mu}Z)} \right|. \end{aligned} \quad (4.10)$$

In this coordinate, the Ricci scalar (2.20) is rewritten as

$$R = -(1 - \eta^2\mu) \frac{1 - \eta^2\mu - (1 + \eta^2\mu) \cosh(4\sqrt{\mu}Z)}{\eta^2\mu - (1 - \eta^2\mu) \sinh^2(2\sqrt{\mu}Z)}. \quad (4.11)$$

In the following, we impose that

$$\eta^2 < \frac{1}{\mu} \quad (4.12)$$

so as to ensure the existence of the undeformed limit.⁹ Note here that this background has a naked singularity at $Z = Z_0$, where

$$Z_0 \equiv \frac{1}{2\sqrt{\mu}} \operatorname{arctanh}(\eta\sqrt{\mu}). \quad (4.13)$$

This is a peculiar feature of the Yang-Baxter deformed geometry based on the modified CYBE like the η -deformation of AdS_5 [46]. From (2.20), in the region with $Z > Z_0$ the Ricci scalar takes negative values, while for $0 < Z < Z_0$, it has positive values (see figure 2). In the undeformed limit $\eta \rightarrow 0$, Z_0 is sent to zero and the singularity disappears because the undeformed spacetime is just AdS_2 . In the following discussion, we focus upon the negative-curvature region ($Z > Z_0$). Therefore, we are concerned with only the upper branch of the potential (4.7).

By performing the following coordinate transformation,

$$r = \frac{1}{\eta} \operatorname{arctanh}(\eta\sqrt{\mu} \coth(2\sqrt{\mu}Z)), \quad (4.14)$$

the metric takes a Schwarzschild-like form¹⁰

$$ds^2 = -4F(r) dT^2 + \frac{dr^2}{F(r)}, \quad (4.15)$$

where the scalar function $F(r)$ is defined as

$$F(r) \equiv \frac{-1 - \eta^2\mu + (1 - \eta^2\mu) \cosh(2\eta r)}{2\eta^2}. \quad (4.16)$$

⁹Otherwise, it is not possible to take the undeformed limit $\eta \rightarrow 0$ because $\eta^2 > 1/\mu$.

¹⁰The factor 4 is included so as to reproduce the result of [28].

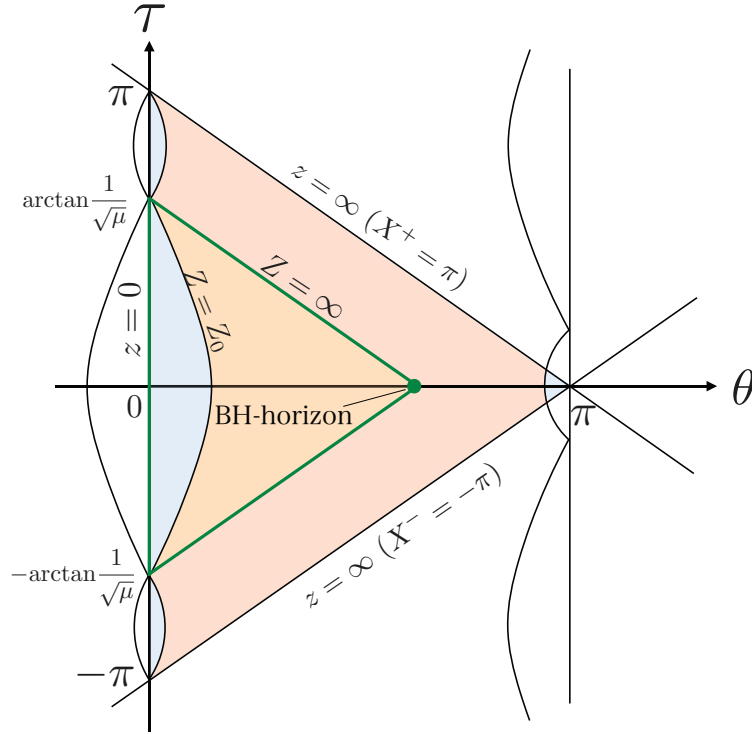


Figure 2. Penrose diagram of the deformed black hole. In this diagram, a curvature singularity is depicted in the global AdS_2 coordinates with $\alpha = 1/2$, $\beta = 0$ and $\gamma = \mu/2$ in (4.3), where τ and θ are the same global coordinates as in the undeformed AdS_2 . The black curves represent the curvature singularities of the deformed spacetime. In the blue region, the scalar curvature is positive, while in the red and orange regions, it takes negative values. The black hole coordinates in (4.10) covers the interior bounded by the green lines. By employing a Schwarzschild-like coordinate system (4.15), we focus on the orange region in order to evaluate the black hole entropy.

In this coordinate system, the dilaton takes the simplest form,

$$\Phi^2 = 1 + r. \quad (4.17)$$

The locations of the boundary and black hole horizon are

$$\text{boundary : } r = \infty, \quad \text{BH horizon : } r = r^* \equiv \frac{1}{\eta} \text{arctanh}(\eta\sqrt{\mu}). \quad (4.18)$$

Bekenstein-Hawking entropy. Let us compute the Bekenstein-Hawking entropy of the deformed black hole by utilizing the coordinate system (4.15).

The Hawking temperature T_H is given by the standard formula:

$$T_H = \frac{1}{4\pi} \partial_r \sqrt{-\frac{g_{tt}}{g_{rr}}} \Big|_{r=r^*} = \frac{\sqrt{\mu}}{\pi}. \quad (4.19)$$

This is the same result as the undeformed case. By assuming that the horizon area A is 1 and using the effective Newton constant G_{eff} in (3.14), the Bekenstein-Hawking entropy

S_{BH} can be computed as

$$S_{\text{BH}} = \frac{A}{4G_{\text{eff}}} \Big|_{r=r^*} = \frac{\text{arctanh}(\pi T_{\text{H}} \eta)}{4G\eta} + \frac{1}{4G}. \quad (4.20)$$

In the undeformed limit $\eta \rightarrow 0$, the entropy is reduced to

$$S_{\text{BH}}^{(\eta=0)} = \frac{\pi T_{\text{H}}}{4G} + \frac{1}{4G},$$

and thus the result of AP model has been reproduced.

5 The boundary computation of entropy

In this section, we compute the entropy of the deformed black hole by evaluating the renormalized boundary stress tensor. Now that the boundary structure is drastically changed, the first thing is to determine the location of the holographic screen. In the following, we take the screen on the singularity by following the proposal of [69–71]. More precisely, by introducing a UV cut-off ϵ , the boundary is taken just before the singularity ($Z = Z_0 + \epsilon$).

In the conformal gauge, the total action including the Gibbons-Hawking term can be rewritten as

$$\begin{aligned} S_{g,\Phi} &= \frac{1}{16\pi G} \int d^2x \sqrt{-g} [\Phi^2 R - \text{U}(\Phi)] + \frac{1}{8\pi G} \int dt \sqrt{-\gamma} \Phi^2 K \\ &= \frac{1}{8\pi G} \int d^2x \left[-4\partial_{(+)}\Phi^2\partial_{(-)}\omega - \frac{1}{2}\text{U}(\Phi)e^{2\omega} \right]. \end{aligned} \quad (5.1)$$

K is the extrinsic curvature and γ is the extrinsic metric. By using the explicit expression of the deformed black hole solution in (4.10), the on-shell bulk action can be evaluated on the boundary,

$$S_{g,\Phi} = \int dt \frac{-\mu}{2\pi G(1 + \eta^2\mu + (-1 + \eta^2\mu)\cosh(4\mu^{\frac{1}{2}}Z))} \Big|_{Z \rightarrow Z_0}. \quad (5.2)$$

Recall that the regulator ϵ is introduced such that $Z - Z_0 = \epsilon$, the on-shell action can be expanded as

$$S_{g,\Phi} = \int dt \left[\frac{1}{16\pi G \eta \epsilon} - \frac{1 + \eta^2\mu}{16\pi G \eta^2} \epsilon^0 + \frac{(3 + \eta^2\mu(-2 + 3\eta^2\mu))\epsilon}{48\pi G \eta^3} + O(\epsilon^2) \right]. \quad (5.3)$$

To cancel the divergence that occurs as the bulk action approaches the boundary, it is appropriate to add the following counter-term:¹¹

$$S_{\text{ct}} = -\frac{1}{8\pi G} \int dt \sqrt{-\gamma_{tt}} \sqrt{F(\Phi^2 - 1) - \frac{1}{\eta^2} \log(1 - \eta^2\mu)}. \quad (5.4)$$

¹¹The dual-theory interpretation of it is not so clear because it contains an infinite number of polynomials and also depends on the temperature explicitly. Another counter-term may be allowed and it would be nice to seek for it by following the procedure in [78]. We are grateful to Ioannis Papadimitriou for this point.

Here the scalar function F is already given in (4.16) and hence

$$F(\Phi^2 - 1) = \frac{-1 - \eta^2\mu + (1 - \eta^2\mu) \cosh(2\eta(\Phi^2 - 1))}{2\eta^2} \quad (5.5)$$

Note that the inside of the root of (5.4) is positive due to the condition (4.12). The extrinsic metric γ_{tt} on the boundary is obtained as

$$\gamma_{tt} = -e^{2\omega}|_{Z \rightarrow Z_0}.$$

In the undeformed limit $\eta \rightarrow 0$, this counter-term reduces to

$$S_{\text{ct}}^{(\eta=0)} = \frac{1}{8\pi G} \int dt \sqrt{-\gamma_{tt}} (1 - \Phi^2), \quad (5.6)$$

because $\Phi^2 - 1 > 0$. This is nothing but the counter-term utilized in the AP model [28].

It is straightforward to check that the sum $S = S_{g,\Phi} + S_{\text{ct}}$ becomes finite on the boundary by using the expanded form of the counter-term (5.4):

$$S_{\text{ct}} = \int dt \left[\frac{-1}{16\pi G \eta \epsilon} + \frac{1 + \eta^2\mu + 2 \log(1 - \eta^2\mu)}{16\pi G \eta^2} \epsilon^0 + O(\epsilon) \right]. \quad (5.7)$$

Around the boundary, the warped factor of the metric in (4.10) can be expanded as

$$e^{2\omega} = \frac{1}{\eta \epsilon} - \left[\frac{1}{\eta^2} + \mu \right] \epsilon^0 + O(\epsilon). \quad (5.8)$$

Hence, by normalizing the boundary metric as

$$\hat{\gamma}_{tt} = \eta \epsilon \gamma_{tt},$$

the boundary stress tensor can be defined as

$$\langle \hat{T}_{tt} \rangle \equiv \frac{-2}{\sqrt{-\hat{\gamma}_{tt}}} \frac{\delta S}{\delta \hat{\gamma}_{tt}} = \lim_{\epsilon \rightarrow 0} \sqrt{\eta \epsilon} \frac{-2}{\sqrt{-\gamma_{tt}}} \frac{\delta S}{\delta \gamma_{tt}}. \quad (5.9)$$

After all, $\langle \hat{T}_{tt} \rangle$ has been evaluated as

$$\langle \hat{T}_{tt} \rangle = -\frac{\log(1 - \eta^2\mu)}{8\pi G \eta^2}. \quad (5.10)$$

To compute the associated entropy, $\langle \hat{T}_{tt} \rangle$ should be identified with energy E like

$$E = -\frac{\log(1 - \pi^2 T_{\text{H}}^2 \eta^2)}{8\pi G \eta^2}, \quad (5.11)$$

where we have used the expression of the Hawking temperature (4.19). Then by solving the thermodynamic relation (3.16) again, the entropy is obtained as

$$S = \frac{\text{arctanh}(\pi T_{\text{H}} \eta)}{4G \eta} + S_{T_{\text{H}}=0}. \quad (5.12)$$

Here $S_{T_H=0}$ has appeared as an integration constant that measures the entropy at zero temperature. Thus the resulting entropy precisely agrees with the Bekenstein-Hawking entropy (4.20), up to the temperature-independent constant.

Finally, it should be remarked that this agreement is quite non-trivial because the deformation changes the UV region of the geometry drastically. Hence the location of the holographic screen and the choice of the counter-term are far from trivial. Although the holographic screen was supposed to be the singularity, inversely speaking, this agreement of the entropies here supports that the proposal in [69–71] would work well. As for the geometrical meaning of the counter-term (5.4), we have no definite idea. It is significant to figure out a systematic prescription to produce the counter-term (5.4).

6 Conclusion and discussion

In this paper, we have discussed deformations of the AP model by following the Yang-Baxter deformation technique. To support the deformed AdS_2 metric, the dilaton itself is deformed and the dilaton potential is also modified from the polynomial to the hyperbolic function-type potential, similarly to integrable deformations. We have obtained the general vacuum solution for the deformed potential.

A particularly interesting example is a deformed black hole solution. The deformation makes the spacetime structure around the boundary change drastically and a new naked singularity appears. The Hawking temperature is the same as in the undeformed case, but the Bekenstein-Hawking entropy is modified due to the deformation. This entropy has also been reproduced by evaluating the renormalized stress tensor with an appropriate counter-term on the regularized screen close to the singularity.

There are some open problems. A possible generalization is to include matter fields, though it has not succeeded yet. The matter contribution would not be so simple in comparison to the AP model. It is also interesting to consider lifting up our results to higher dimensional setups. Possibly, the most intriguing issue is to clarify the dual quantum mechanics for the deformed black hole presented here. A candidate would be a deformed SYK model which would be constructed by performing a disordered quench for a q -deformed Heisenberg magnet. When an infinitesimal deformation of the deformed AdS_2 geometry is considered, one would encounter a deformed Schwarzian derivative, though it seems difficult to determine what it is because there is no $\text{SL}(2)$ invariance on the boundary in comparison to the standard setup studied in [12, 29, 30]. It is also interesting to study Yang-Baxter deformations of the Callan-Giddings-Harvey-Strominger (CGHS) model [79] by following [72, 80, 81].

There are some future directions associated with Yang-Baxter deformations as well. Now that we know the classical r -matrix which leads to the black hole geometry, it would be interesting to consider a Yang-Baxter deformation of higher-dimensional AdS with this r -matrix. In the study of Yang-Baxter deformations, it has been a long standing problem to determine where the holographic screen is, while there was a proposal for the η -deformed AdS_5 [69–71] but it has not been supported by concrete evidence before this paper. It is

significant to find out more supports to clarify the holographic interpretation for general Yang-Baxter deformations.

We hope that the deformed AP model would provide a new arena to study the correspondence between nearly AdS_2 geometries and 1D quantum mechanical system like the SYK model or its cousins.

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References

- [1] G. 't Hooft, *Dimensional reduction in quantum gravity*, [gr-qc/9310026](#) [[INSPIRE](#)].
- [2] L. Susskind, *The world as a hologram*, *J. Math. Phys.* **36** (1995) 6377 [[hep-th/9409089](#)] [[INSPIRE](#)].
- [3] R. Bousso, *The holographic principle*, *Rev. Mod. Phys.* **74** (2002) 825 [[hep-th/0203101](#)] [[INSPIRE](#)].
- [4] J.M. Maldacena, *The large- N limit of superconformal field theories and supergravity*, *Int. J. Theor. Phys.* **38** (1999) 1113 [[hep-th/9711200](#)] [[INSPIRE](#)].
- [5] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from noncritical string theory*, *Phys. Lett. B* **428** (1998) 105 [[hep-th/9802109](#)] [[INSPIRE](#)].
- [6] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [[hep-th/9802150](#)] [[INSPIRE](#)].
- [7] N. Beisert et al., *Review of AdS/CFT Integrability: An Overview*, *Lett. Math. Phys.* **99** (2012) 3 [[arXiv:1012.3982](#)] [[INSPIRE](#)].
- [8] A. Kitaev, *A simple model of quantum holography*, talks at *KITP*, 7 April and 27 May 2015, [<http://online.kitp.ucsb.edu/online/entangled15/kitaev/>] [<http://online.kitp.ucsb.edu/online/entangled15/kitaev2/>].
- [9] S. Sachdev and J. Ye, *Gapless spin fluid ground state in a random, quantum Heisenberg magnet*, *Phys. Rev. Lett.* **70** (1993) 3339 [[cond-mat/9212030](#)] [[INSPIRE](#)].
- [10] J. Polchinski and V. Rosenhaus, *The Spectrum in the Sachdev-Ye-Kitaev Model*, *JHEP* **04** (2016) 001 [[arXiv:1601.06768](#)] [[INSPIRE](#)].

- [11] J. Maldacena and D. Stanford, *Remarks on the Sachdev-Ye-Kitaev model*, *Phys. Rev. D* **94** (2016) 106002 [[arXiv:1604.07818](#)] [[INSPIRE](#)].
- [12] K. Jensen, *Chaos in AdS_2 Holography*, *Phys. Rev. Lett.* **117** (2016) 111601 [[arXiv:1605.06098](#)] [[INSPIRE](#)].
- [13] D.J. Gross and V. Rosenhaus, *A Generalization of Sachdev-Ye-Kitaev*, *JHEP* **02** (2017) 093 [[arXiv:1610.01569](#)] [[INSPIRE](#)].
- [14] W. Fu, D. Gaiotto, J. Maldacena and S. Sachdev, *Supersymmetric Sachdev-Ye-Kitaev models*, *Phys. Rev. D* **95** (2017) 026009 [[arXiv:1610.08917](#)] [[INSPIRE](#)].
- [15] E. Witten, *An SYK-Like Model Without Disorder*, [arXiv:1610.09758](#) [[INSPIRE](#)].
- [16] J.S. Cotler et al., *Black Holes and Random Matrices*, [arXiv:1611.04650](#) [[INSPIRE](#)].
- [17] I.R. Klebanov and G. Tarnopolsky, *Uncolored random tensors, melon diagrams and the Sachdev-Ye-Kitaev models*, *Phys. Rev. D* **95** (2017) 046004 [[arXiv:1611.08915](#)] [[INSPIRE](#)].
- [18] T. Nishinaka and S. Terashima, *A Note on Sachdev-Ye-Kitaev Like Model without Random Coupling*, [arXiv:1611.10290](#) [[INSPIRE](#)].
- [19] A. Larkin and Y.N. Ovchinnikov, *Quasiclassical method in the theory of superconductivity*, *JETP* **28** (1969) 1200.
- [20] A. Kitaev, *Correlations in the Hawking Radiation and Thermal Noise*, talk given at *Fundamental Physics Prize Symposium*, 10 November 2014.
- [21] J. Maldacena, S.H. Shenker and D. Stanford, *A bound on chaos*, *JHEP* **08** (2016) 106 [[arXiv:1503.01409](#)] [[INSPIRE](#)].
- [22] S.H. Shenker and D. Stanford, *Black holes and the butterfly effect*, *JHEP* **03** (2014) 067 [[arXiv:1306.0622](#)] [[INSPIRE](#)].
- [23] S.H. Shenker and D. Stanford, *Stringy effects in scrambling*, *JHEP* **05** (2015) 132 [[arXiv:1412.6087](#)] [[INSPIRE](#)].
- [24] J. Polchinski, *Chaos in the black hole S-matrix*, [arXiv:1505.08108](#) [[INSPIRE](#)].
- [25] R. Jackiw, *Lower Dimensional Gravity*, *Nucl. Phys. B* **252** (1985) 343 [[INSPIRE](#)].
- [26] C. Teitelboim, *Gravitation and Hamiltonian Structure in Two Space-Time Dimensions*, *Phys. Lett. B* **126** (1983) 41 [[INSPIRE](#)].
- [27] D. Grumiller, W. Kummer and D.V. Vassilevich, *Dilaton gravity in two-dimensions*, *Phys. Rept.* **369** (2002) 327 [[hep-th/0204253](#)] [[INSPIRE](#)].
- [28] A. Almheiri and J. Polchinski, *Models of AdS_2 backreaction and holography*, *JHEP* **11** (2015) 014 [[arXiv:1402.6334](#)] [[INSPIRE](#)].
- [29] J. Maldacena, D. Stanford and Z. Yang, *Conformal symmetry and its breaking in two dimensional Nearly Anti-de-Sitter space*, *PTEP* **2016** (2016) 12C104 [[arXiv:1606.01857](#)] [[INSPIRE](#)].
- [30] J. Engelsöy, T.G. Mertens and H. Verlinde, *An investigation of AdS_2 backreaction and holography*, *JHEP* **07** (2016) 139 [[arXiv:1606.03438](#)] [[INSPIRE](#)].
- [31] C. Klimčík, *Yang-Baxter σ -models and dS/AdS T duality*, *JHEP* **12** (2002) 051 [[hep-th/0210095](#)] [[INSPIRE](#)].
- [32] C. Klimčík, *On integrability of the Yang-Baxter σ -model*, *J. Math. Phys.* **50** (2009) 043508 [[arXiv:0802.3518](#)] [[INSPIRE](#)].
- [33] F. Delduc, M. Magro and B. Vicedo, *On classical q -deformations of integrable σ -models*, *JHEP* **11** (2013) 192 [[arXiv:1308.3581](#)] [[INSPIRE](#)].

- [34] T. Matsumoto and K. Yoshida, *Yang-Baxter σ -models based on the CYBE*, *Nucl. Phys. B* **893** (2015) 287 [[arXiv:1501.03665](#)] [[INSPIRE](#)].
- [35] I. Kawaguchi and K. Yoshida, *Hybrid classical integrability in squashed σ -models*, *Phys. Lett. B* **705** (2011) 251 [[arXiv:1107.3662](#)] [[INSPIRE](#)].
- [36] I. Kawaguchi and K. Yoshida, *Hidden Yangian symmetry in σ -model on squashed sphere*, *JHEP* **11** (2010) 032 [[arXiv:1008.0776](#)] [[INSPIRE](#)].
- [37] I. Kawaguchi, T. Matsumoto and K. Yoshida, *The classical origin of quantum affine algebra in squashed σ -models*, *JHEP* **04** (2012) 115 [[arXiv:1201.3058](#)] [[INSPIRE](#)].
- [38] I. Kawaguchi, T. Matsumoto and K. Yoshida, *On the classical equivalence of monodromy matrices in squashed σ -model*, *JHEP* **06** (2012) 082 [[arXiv:1203.3400](#)] [[INSPIRE](#)].
- [39] F. Delduc, T. Kameyama, M. Magro and B. Vicedo, *Affine q -deformed symmetry and the classical Yang-Baxter σ -model*, [arXiv:1701.03691](#) [[INSPIRE](#)].
- [40] I. Kawaguchi and K. Yoshida, *Classical integrability of Schrödinger σ -models and q -deformed Poincaré symmetry*, *JHEP* **11** (2011) 094 [[arXiv:1109.0872](#)] [[INSPIRE](#)].
- [41] I. Kawaguchi and K. Yoshida, *Exotic symmetry and monodromy equivalence in Schrödinger σ -models*, *JHEP* **02** (2013) 024 [[arXiv:1209.4147](#)] [[INSPIRE](#)].
- [42] I. Kawaguchi, T. Matsumoto and K. Yoshida, *Schroedinger σ -models and Jordanian twists*, *JHEP* **08** (2013) 013 [[arXiv:1305.6556](#)] [[INSPIRE](#)].
- [43] F. Delduc, M. Magro and B. Vicedo, *An integrable deformation of the $AdS_5 \times S^5$ superstring action*, *Phys. Rev. Lett.* **112** (2014) 051601 [[arXiv:1309.5850](#)] [[INSPIRE](#)].
- [44] F. Delduc, M. Magro and B. Vicedo, *Derivation of the action and symmetries of the q -deformed $AdS_5 \times S^5$ superstring*, *JHEP* **10** (2014) 132 [[arXiv:1406.6286](#)] [[INSPIRE](#)].
- [45] I. Kawaguchi, T. Matsumoto and K. Yoshida, *Jordanian deformations of the $AdS_5 \times S^5$ superstring*, *JHEP* **04** (2014) 153 [[arXiv:1401.4855](#)] [[INSPIRE](#)].
- [46] G. Arutyunov, R. Borsato and S. Frolov, *S -matrix for strings on η -deformed $AdS_5 \times S^5$* , *JHEP* **04** (2014) 002 [[arXiv:1312.3542](#)] [[INSPIRE](#)].
- [47] G. Arutyunov, R. Borsato and S. Frolov, *Puzzles of η -deformed $AdS_5 \times S^5$* , *JHEP* **12** (2015) 049 [[arXiv:1507.04239](#)] [[INSPIRE](#)].
- [48] B. Hoare, R. Roiban and A.A. Tseytlin, *On deformations of $AdS_n \times S^n$ supercosets*, *JHEP* **06** (2014) 002 [[arXiv:1403.5517](#)] [[INSPIRE](#)].
- [49] O. Lunin, R. Roiban and A.A. Tseytlin, *Supergravity backgrounds for deformations of $AdS_n \times S^n$ supercoset string models*, *Nucl. Phys. B* **891** (2015) 106 [[arXiv:1411.1066](#)] [[INSPIRE](#)].
- [50] B. Hoare and A.A. Tseytlin, *Type IIB supergravity solution for the T -dual of the η -deformed $AdS_5 \times S^5$ superstring*, *JHEP* **10** (2015) 060 [[arXiv:1508.01150](#)] [[INSPIRE](#)].
- [51] G. Arutyunov, S. Frolov, B. Hoare, R. Roiban and A.A. Tseytlin, *Scale invariance of the η -deformed $AdS_5 \times S^5$ superstring, T -duality and modified type-II equations*, *Nucl. Phys. B* **903** (2016) 262 [[arXiv:1511.05795](#)] [[INSPIRE](#)].
- [52] L. Wulff and A.A. Tseytlin, *κ -symmetry of superstring σ -model and generalized 10d supergravity equations*, *JHEP* **06** (2016) 174 [[arXiv:1605.04884](#)] [[INSPIRE](#)].
- [53] T. Matsumoto and K. Yoshida, *Lunin-Maldacena backgrounds from the classical Yang-Baxter equation — towards the gravity/CYBE correspondence*, *JHEP* **06** (2014) 135 [[arXiv:1404.1838](#)] [[INSPIRE](#)].

- [54] T. Matsumoto and K. Yoshida, *Integrability of classical strings dual for noncommutative gauge theories*, *JHEP* **06** (2014) 163 [[arXiv:1404.3657](#)] [[INSPIRE](#)].
- [55] T. Matsumoto and K. Yoshida, *Schrödinger geometries arising from Yang-Baxter deformations*, *JHEP* **04** (2015) 180 [[arXiv:1502.00740](#)] [[INSPIRE](#)].
- [56] I. Kawaguchi, T. Matsumoto and K. Yoshida, *A Jordanian deformation of AdS space in type IIB supergravity*, *JHEP* **06** (2014) 146 [[arXiv:1402.6147](#)] [[INSPIRE](#)].
- [57] S.J. van Tongeren, *On classical Yang-Baxter based deformations of the $AdS_5 \times S^5$ superstring*, *JHEP* **06** (2015) 048 [[arXiv:1504.05516](#)] [[INSPIRE](#)].
- [58] S.J. van Tongeren, *Yang-Baxter deformations, AdS/CFT and twist-noncommutative gauge theory*, *Nucl. Phys. B* **904** (2016) 148 [[arXiv:1506.01023](#)] [[INSPIRE](#)].
- [59] T. Matsumoto and K. Yoshida, *Yang-Baxter deformations and string dualities*, *JHEP* **03** (2015) 137 [[arXiv:1412.3658](#)] [[INSPIRE](#)].
- [60] T. Kameyama, H. Kyono, J.-i. Sakamoto and K. Yoshida, *Lax pairs on Yang-Baxter deformed backgrounds*, *JHEP* **11** (2015) 043 [[arXiv:1509.00173](#)] [[INSPIRE](#)].
- [61] H. Kyono and K. Yoshida, *Supercoset construction of Yang-Baxter deformed $AdS_5 \times S^5$ backgrounds*, *PTEP* **2016** (2016) 083B03 [[arXiv:1605.02519](#)] [[INSPIRE](#)].
- [62] B. Hoare and S.J. van Tongeren, *On Jordanian deformations of AdS_5 and supergravity*, *J. Phys. A* **49** (2016) 434006 [[arXiv:1605.03554](#)] [[INSPIRE](#)].
- [63] D. Orlando, S. Reffert, J.-i. Sakamoto and K. Yoshida, *Generalized type IIB supergravity equations and non-Abelian classical r-matrices*, *J. Phys. A* **49** (2016) 445403 [[arXiv:1607.00795](#)] [[INSPIRE](#)].
- [64] R. Borsato and L. Wulff, *Target space supergeometry of η and λ -deformed strings*, *JHEP* **10** (2016) 045 [[arXiv:1608.03570](#)] [[INSPIRE](#)].
- [65] D. Osten and S.J. van Tongeren, *Abelian Yang-Baxter deformations and TsT transformations*, *Nucl. Phys. B* **915** (2017) 184 [[arXiv:1608.08504](#)] [[INSPIRE](#)].
- [66] B. Hoare and A.A. Tseytlin, *Homogeneous Yang-Baxter deformations as non-abelian duals of the AdS_5 σ -model*, *J. Phys. A* **49** (2016) 494001 [[arXiv:1609.02550](#)] [[INSPIRE](#)].
- [67] R. Borsato and L. Wulff, *Integrable Deformations of T-Dual σ Models*, *Phys. Rev. Lett.* **117** (2016) 251602 [[arXiv:1609.09834](#)] [[INSPIRE](#)].
- [68] S.J. van Tongeren, *Almost abelian twists and AdS/CFT*, *Phys. Lett. B* **765** (2017) 344 [[arXiv:1610.05677](#)] [[INSPIRE](#)].
- [69] T. Kameyama and K. Yoshida, *A new coordinate system for q-deformed $AdS_5 \times S^5$ and classical string solutions*, *J. Phys. A* **48** (2015) 075401 [[arXiv:1408.2189](#)] [[INSPIRE](#)].
- [70] T. Kameyama and K. Yoshida, *Minimal surfaces in q-deformed $AdS_5 \times S^5$ with Poincaré coordinates*, *J. Phys. A* **48** (2015) 245401 [[arXiv:1410.5544](#)] [[INSPIRE](#)].
- [71] T. Kameyama and K. Yoshida, *Generalized quark-antiquark potentials from a q-deformed $AdS_5 \times S^5$ background*, *PTEP* **2016** (2016) 063B01 [[arXiv:1602.06786](#)] [[INSPIRE](#)].
- [72] A. Borowiec, H. Kyono, J. Lukierski, J.-i. Sakamoto and K. Yoshida, *Yang-Baxter σ -models and Lax pairs arising from κ -Poincaré r-matrices*, *JHEP* **04** (2016) 079 [[arXiv:1510.03083](#)] [[INSPIRE](#)].
- [73] C.R. Nappi and E. Witten, *A WZW model based on a nonsemisimple group*, *Phys. Rev. Lett.* **71** (1993) 3751 [[hep-th/9310112](#)] [[INSPIRE](#)].

- [74] H. Kyono and K. Yoshida, *Yang-Baxter invariance of the Nappi-Witten model*, *Nucl. Phys. B* **905** (2016) 242 [[arXiv:1511.00404](#)] [[INSPIRE](#)].
- [75] N. Ikeda and K.I. Izawa, *General form of dilaton gravity and nonlinear gauge theory*, *Prog. Theor. Phys.* **90** (1993) 237 [[hep-th/9304012](#)] [[INSPIRE](#)].
- [76] B. Vicedo, *Deformed integrable σ -models, classical R -matrices and classical exchange algebra on Drinfel'd doubles*, *J. Phys. A* **48** (2015) 355203 [[arXiv:1504.06303](#)] [[INSPIRE](#)].
- [77] B. Hoare and S.J. van Tongeren, *Non-split and split deformations of AdS_5* , *J. Phys. A* **49** (2016) 484003 [[arXiv:1605.03552](#)] [[INSPIRE](#)].
- [78] M. Cvetič and I. Papadimitriou, *AdS_2 holographic dictionary*, *JHEP* **12** (2016) 008 [Erratum *ibid.* **01** (2017) 120] [[arXiv:1608.07018](#)] [[INSPIRE](#)].
- [79] C.G. Callan Jr., S.B. Giddings, J.A. Harvey and A. Strominger, *Evanescent black holes*, *Phys. Rev. D* **45** (1992) R1005 [[hep-th/9111056](#)] [[INSPIRE](#)].
- [80] T. Matsumoto, D. Orlando, S. Reffert, J.-i. Sakamoto and K. Yoshida, *Yang-Baxter deformations of Minkowski spacetime*, *JHEP* **10** (2015) 185 [[arXiv:1505.04553](#)] [[INSPIRE](#)].
- [81] H. Kyono, J.-i. Sakamoto and K. Yoshida, *Lax pairs for deformed Minkowski spacetimes*, *JHEP* **01** (2016) 143 [[arXiv:1512.00208](#)] [[INSPIRE](#)].